Solutions to Numerical Problems

First Mid-term test

1. Given $d = 15\,mm$

$\tau_{min} = 0$

$\tau_{max} = 35\,MPa$

$\sigma_{min} = -15\,MPa$

$\sigma_{max} = 30\,MPa$

$\sigma_u = 540\,MPa$

$\sigma_y = 400\,MPa$

$\sigma_n = 200\,MPa$

$\tau_n = 160\,MPa.$

We need to determine factor of safety.

From the given data,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 7.5\,MPa$$

$$\sigma_{av} = \frac{\sigma_{max} - \sigma_{min}}{2} = 22.5\,MPa$$

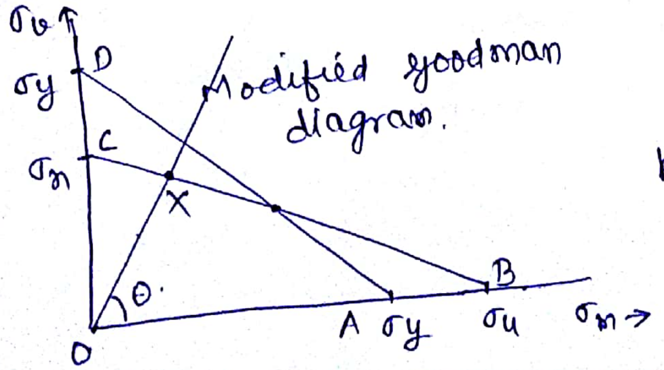
$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = 17.5\,MPa$$

$$\tau_v = \frac{\tau_{max} - \tau_{min}}{2} = 17.5\,MPa.$$

From the max. distortion energy theory

$$\text{equivalent } \sigma_m = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{(7.5)^2 + 3(17.5)^2}$$
$$= 31.22\,MPa$$

$$\text{Equivalent } \sigma_v = \sqrt{\sigma_{av}^2 + 3\tau_v^2} = \sqrt{(22.5)^2 + 3(17.5)^2}$$
$$= 37.74\,MPa.$$



Modified goodman diagram.

$$\tan\theta = \frac{\sigma_v}{\sigma_m} \quad \text{or} \quad \theta = 50.4^\circ \quad \text{---(i)}$$

pt. X lies on line CB hence

$$\frac{\sigma_m}{540} + \frac{\sigma_v}{200} = 1 \quad \text{-----(ii)}$$

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Solving eq. (i) and (ii) simultaneously

Limiting mean stress $\sigma_m = 126.74$ MPa

Limiting variable stress $\sigma_v = 153.10$ MPa

$$f_s = \frac{\text{Limiting stress}}{\text{Actual stress}} = \frac{153.10}{37.74} = 4.05.$$

So factor of safety for the given combined load condition is 4.05. Ans

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3. Given $\psi = 30°$; $P = 35$ kW

$N = 2000$ rpm ; $T_p = 25$

$\phi = 20°$; $\sigma_b = 100$ MPa

$b = 3$ pn ; $C_v = \dfrac{6}{6+v}$.

$$y' = 0.154 - \frac{0.912}{T_e}$$

Beam strength of helical gear by lewis eq.

$F_b = \sigma_b \cdot b \cdot y' \cdot p_n = \sigma_b \cdot b \cdot y' \times p_c \cos\psi$.

virtual / eq. no. of teeth on pinion $= \dfrac{T_p}{\cos^3\psi} = 38.49$

Lewis form factor for virtual teeth $= 0.154 - \dfrac{0.912}{38.49} = 0.130$

so Beam strength $F_b = 100 \times 3m \times 0.130 \times 3.14 \times m \times \cos 30°$

$F_b = 288.39\ m^2$.

assuming $v = 15$ m/sec

$C_v = 0.285$

$$F_{eff} = \frac{C_s \cdot F_t}{C_v}$$

assuming service factor as 1.5

$$F_{eff} = \frac{1.5 \times 2T}{C_v \times d} = \frac{1.5}{C_v} \times \frac{2 \times P \times 60 \times 1000}{d \times 2\pi N} = \frac{221052}{m}.$$

assuming $f_s = 1.5$, for safe design $F_b = f_s \cdot F_{eff}$

or $288.39\ m^2 = 1.5 \times \dfrac{221052}{m}$ or $m = 10.47$

$\simeq 12$ mm

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so module $= 12\,mm$

actual velocity $= \dfrac{\pi \times 12 \times 25 \times 2000}{60,000} = 31.4\ m/sec.$

Pitch circle dia. $= m.T = 12 \times 25 = 300\ mm.$

Ans

Second Mid-term test

1. Given $F_{min} = 90\,N$ deflection for 45N load $= 7.5\,mm.$

$F_{max} = 135\,N$

spring index $C = \frac{D}{d} = 10$

$\tau_{per} = 480\,MPa$ & $G = 80\,GPa.$

we know $\tau_{max} = K \cdot \dfrac{8 F_{max} D}{\pi d^3} = \dfrac{K \cdot 8 F_{max} C}{\pi d^2}.$

Wahl's stress factor $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} = \dfrac{39}{36} + \dfrac{0.615}{10} = 1.144$

putting all the values,

$$480 = \frac{1.14 \times 8 \times 135 \times 10}{\pi d^2}$$

or $d^2 = 8.168$ or $d = 2.85\ mm.$

hence $D = 2.85\,C = 28.5\ mm.$

No. of turns -

we know $\delta = \dfrac{8 F D^3 n}{G d^4} = \dfrac{8 F C^3 n}{G \cdot d}$

or $n = \dfrac{\delta \cdot G \cdot d}{8 F C^3} = \dfrac{7.5 \times 80 \times 10^3 \times 2.85}{8 \times 45 \times 1000}$

$n = 4.75 \simeq 5$

solid length $L_s = n \cdot d = 5 \times 2.85 = 14.25\ mm$

Free length $L_F = L_s + \delta_{max} + 0.15\,\delta_{max} = 22.87\ mm.$

Ans

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2. Given $F_r = 8$ kN ; $F_a = 3$ kN

$N = 1200$ rpm ; $L_h = 20{,}000$ hrs.

shaft dia. = 75 mm. ; $C_s = 1.0$.

since $\frac{F_a}{F_r} = 0.375 < 0.7$ so single row deep groove ball bearing will suffice.

Probable bearing can be - 6015, 6215, 6315, 6415.

For bearing 6315, values of X and Y from catalogue are - $X = \cancel{100}\ 0.56$ and $Y = 1.5$.

so eq. load $P = (X F_r + Y F_a) \cdot C_s$

$= 8980$ N.

$$L_{10} = \frac{60 \times N \times L_{10h}}{10^6} = \frac{60 \times 1200 \times 20000}{10^6} = 1440 \text{ m rev.}$$

$$C = P(L_{10})^{1/3} = 8980\,(1440)^{1/3} = 101406 \text{ N.}$$

since dynamic load carrying capacity C for bearing 6315 is more than required value of 101406 N, hence bearing <u>6315</u> is suitable for given application. <u>Ans</u>

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3. Given $F_r = 25$ kN ; $N = 900$ rpm.

bearing $p_0 = 2.5$ MPa ; $L/D = 1.0$.

viscosity $= 2.5$ cp

since $p_0 = \frac{W}{L \times D}$ $\therefore$ $2.5 = \frac{25 \times 10^3}{D^2}$

or $D = 100$ mm & $L = 100$ mm.

we know sommerfield no.

$$S = \left(\frac{D}{C}\right)^2 \times \frac{Z n}{p}$$

Z is viscosity in MPa-s

$= \frac{Z_{cp}}{10^9} = 2.5 \times 10^{-9}$

n is rev/sec

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assuming diametral clearance ratio as 1000,

$$S = (1000)^2 \times \frac{2.5\times10^{-9} \times 900}{2.5 \times 60} = 0.015$$

from Raymondi & Boyd charts, for $L/D = 1.0$ & $S = 0.015$

$\mu \frac{D}{c} = 1.0$ hence coefficient of friction $= 0.001$

$\frac{h_o}{c} = 0.08$ hence min. film thickness

$h_o = 0.08 \times c$

$\cancel{= 0.08 \times \frac{1000}{D}} = \frac{\cancel{800}}{\cancel{100}}$

$= 0.08 \times \frac{D}{1000} = 0.08 \times \frac{100}{1000}$

$= 0.008$ mm.

Heat generated $H_g = \mu W.v$ watts

$= 0.001 \times 25\times10^3 \times \frac{\pi \times 100 \times 900}{60,000}$

$= 117.75$ watts.

Head dissipated $= \frac{(\Delta t + 18)^2 LD}{K}$

assuming $K = 0.273$ W/m^2 °C

& $\Delta t = t_b - t_a = \frac{1}{2}(t_o - t_a) = \frac{1}{2}(75 - 25) = 25$°C

$H_d = \frac{(25+18)^2 \times 0.1 \times 0.1}{0.273} = 67.72$ watts

so amt. of artificial cooling req. $= H_g - H_d$

$= 50$ watts.

from charts, flow variable $\frac{Q}{DCn'L} = 4.8$

$\therefore Q = 4.8 \times 0.1 \times 0.0.0001 \times \frac{900}{60} \times 0.1$ m^3/sec

$Q = 7.2\times10^{-5}$ m^3/sec $= 7200$ mm^3/sec **Ans**

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4. Given $P = 20\,kW$; $N = 480\,rpm$

$n = 1440\,rpm$; op. = 15 hr/day.

$C = 1.2$ metres

For the given power C type V-belt might be suitable.

For C type belt min. pitch dia. $d = 225\,mm$.

hence Larger pulley Dia. D

$$= \frac{225 \times 1440}{480}$$

$$= 675\,mm.$$

Nominal pitch length

$$L_p = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C}$$

$$L_p = 2 \times 1.2 + \frac{\pi(0.675 + 0.225)}{2} + \frac{(0.675 + 0.225)^2}{4 \times 1.2}$$

$$L_p = 2.4 + 1.413 + 0.1687 = 3.98\,m$$

$$= 3981\,mm$$

corresponding pitch length from data book $= 3713\,mm.$

& corresponding inside length $= 3658\,mm.$

so belt is C 3658

$$\text{No. of belt} = \frac{\text{Design power}}{\text{power rating of one belt}}$$

$$\text{Design power} = \frac{\text{Rated power} \times \text{service factor}}{\text{Corr. factor for length} \times \text{Corr. factor for AOC}}$$

$k_l = 0.99$ & $k_\theta = 0.94$ & $k_s = 1.3$

$$\text{design power} = \frac{20 \times 1.3}{0.99 \times 0.94} = 27.93\,kW.$$

Power rating of one belt $= 9.03\,kW$

hence no. of belt $= \frac{27.93}{9.03} = 3.09 \simeq 4$ belts **Ans**

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5. Given thickness = 8 mm

width = 100 mm

v = 1600 m/min = 26.66 m/sec

$\dot{m}$ = 0.9 kg/m.

θ = 165°

μ = 0.3

σ = 2 MPa.

Centrifugal tension $T_c = \dot{m} v^2$

$= 0.9 \times (26.66)^2$

$= 640 N.$

For Max. Power, Max. Tension $= 3T_c$

$= 1920 N.$

So $T_1 = T - T_c = 1920 - 640 = 1280 N.$

& $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 165° \times \frac{\pi}{180}} = 2.37.$

$\therefore T_2 = \frac{T_1}{2.37} = 539.75 N.$

hence power transmitted $= (T_1 - T_2) v$

$= (1280 - 539.75) 26.66$

$= 19.73 kW$

Initial tension in belt

$T_0 = \frac{T_1 + T_2 + 2T_c}{2}$

$= \frac{1280 + 539.75 + 2 \times 640}{2}$

$= 1549.875 N.$ Ans

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