**Department of Mechanical Engineering**

**VII Semester (12/9/2017)**

**I - Test**

**Subject- Operations Research**

**Time-1 Hr.** **M.M.- 10**

Consider following LPP:

Maximize $\mathbf{Z = - x_1 + 2x_2}$

Subject to

$$\mathbf{-x_1 + x_2 \leq 1}$$
$$\mathbf{x_1 + x_2 \leq 2}$$
$$\mathbf{x_1, x_2 \geq 0}$$

(i) Solve above LPP using the graphical method. (1)

(ii) Solve above LPP using the Simplex method. (2)

(iii) Using optimal table obtained in (ii), find new optimal solution when objective function is $\mathbf{2x_1 + x_2}$ (2)

(iv) Using optimal table obtained in (ii), find optimal solution when RHS of the constraints changes from **[1, 2]** to **[2, 1]**. (1)

(v) Write the dual of the above LPP. (1)

(vi) Using optimal table obtained in (ii), write the values of dual variables and discuss their economic interpretation. (1)

(vii) Solve above LPP when variables are integers using branch & bound method. (2)

**Department of Mechanical Engineering**

**VII Semester (7/11/2017)**

**II test**

**Subject- Operations Research**

**Time-1 Hr.** **M.M.- 10**

(1). Solve the following transportation problem: (3)

| | W1 | W2 | W3 | W4 | **Availability** |
|---|---|---|---|---|---|
| F1 | 15 | 10 | 17 | 18 | **2** |
| F2 | 16 | 13 | 12 | 13 | **6** |
| F3 | 12 | 17 | 20 | 11 | **7** |
| **Demand** | **3** | **3** | **4** | **5** | |

(2). Solve the following assignment problem: (2)

| | I | II | III | IV |
|---|---|---|---|---|
| A | 5 | 3 | 1 | 8 |
| B | 7 | 9 | 2 | 6 |
| C | 6 | 4 | 5 | 7 |
| D | 5 | 7 | 7 | 6 |

(3). Solve the following games: (2)

| | | Player B | | | |
|---|---|---|---|---|---|
| | | B1 | B2 | B3 | B4 |
| Player A | A1 | 3 | 2 | 4 | 0 |
| | A2 | 3 | 4 | 2 | 4 |
| | A3 | 4 | 2 | 4 | 0 |
| | A4 | 0 | 4 | 0 | 4 |

| | | Player B | | | |
|---|---|---|---|---|---|
| | | B1 | B2 | B3 | B4 |
| Player A | A1 | 2 | 1 | 0 | – 2 |
| | A2 | 1 | 0 | 3 | 2 |

(4). At present a company is purchasing an item X from outside suppliers. The consumption is 10,000 units/year (250 working days). The cost of the item is Rs 5 per unit and the ordering cost is estimated to be Rs 100 per order. The cost of carrying inventory is 25%. If the consumption rate is uniform, determine the economic purchasing quantity.

In the above problem assume the company is going to manufacture the item with the equipment that is estimated to produce 100 units per day. The cost of the unit thus produced is Rs 3.50 per unit. The set-up cost is Rs 150 per set up and the inventory carrying charge is 25%. Explain whether you would purchase or manufacture. (3)

Consider following LPP:

Maximize $\mathbf{Z = - x_1 + 2x_2}$

Subject to

$$\mathbf{-x_1 + x_2 \leq 1}$$

$$\mathbf{x_1 + x_2 \leq 2}$$

$$\mathbf{x_1, x_2 \geq 0}$$

i. Solve above LPP using the graphical method.

ii. Solve above LPP using the Simplex method.

iii. Using optimal table obtained in (ii), find new optimal solution when objective function is $2x_1 + x_2$

iv. Using optimal table obtained in (ii), find optimal solution when RHS of the constraints changes from [1, 2] to [2, 1].

v. Write the dual of the above LPP.

vi. Using optimal table obtained in (ii), write the values of dual variables and discuss their economic interpretation.

vii. Solve above LPP when variables are integers using branch & bound method.

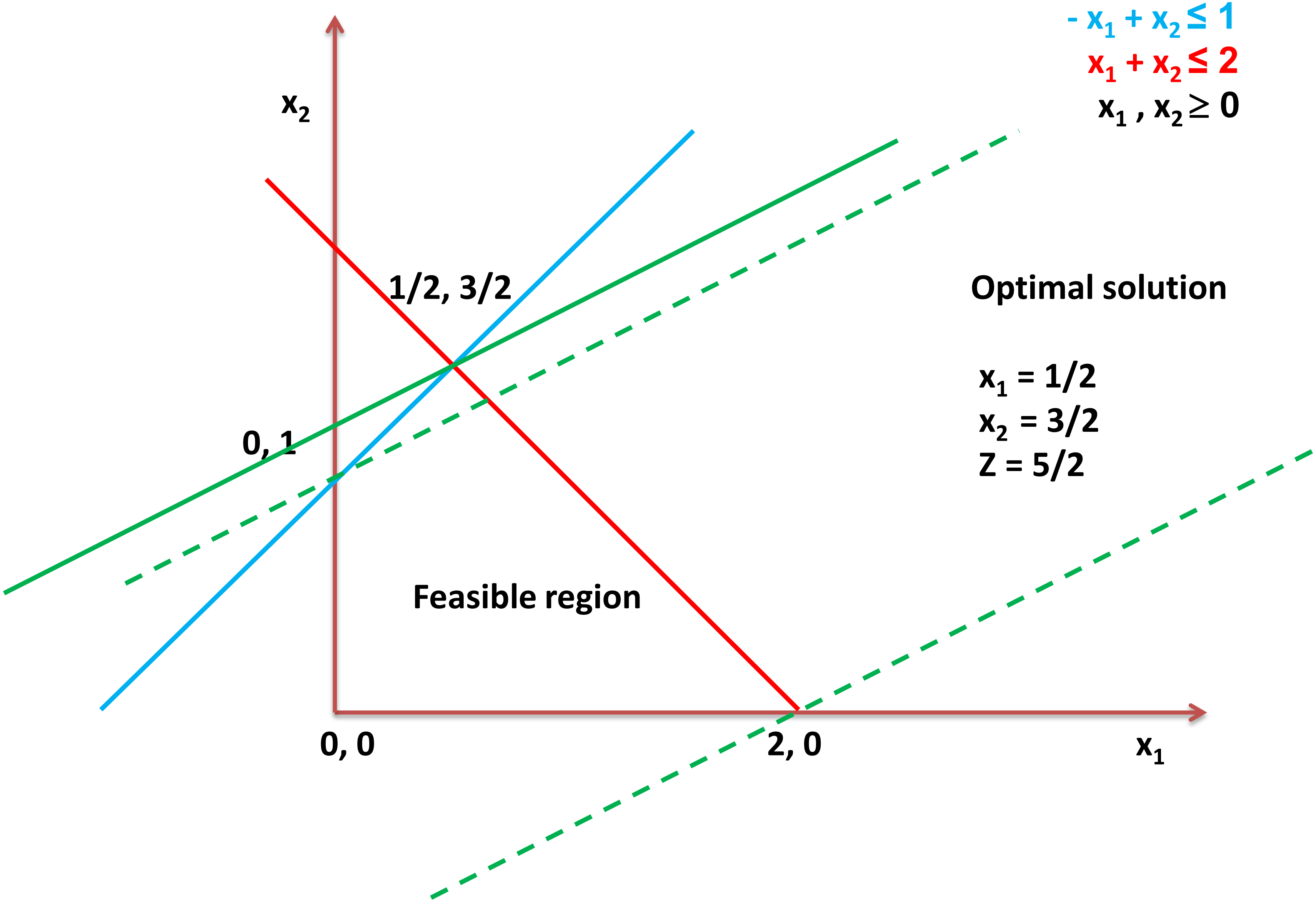
$$\text{Max } Z = -x_1 + 2x_2$$

Subject to

$$-x_1 + x_2 \leq 1$$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$



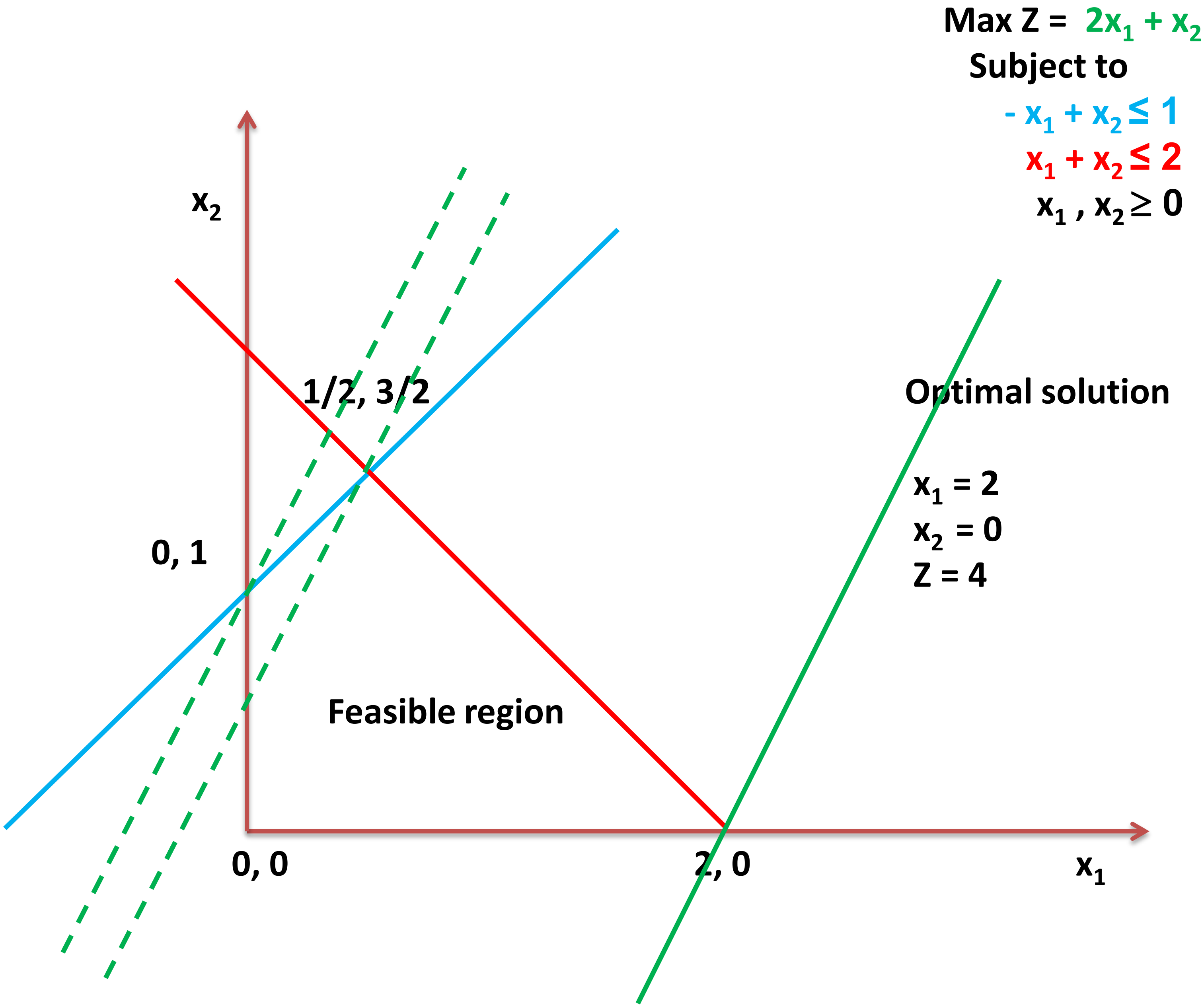

Optimal solution

$x_1 = 1/2$

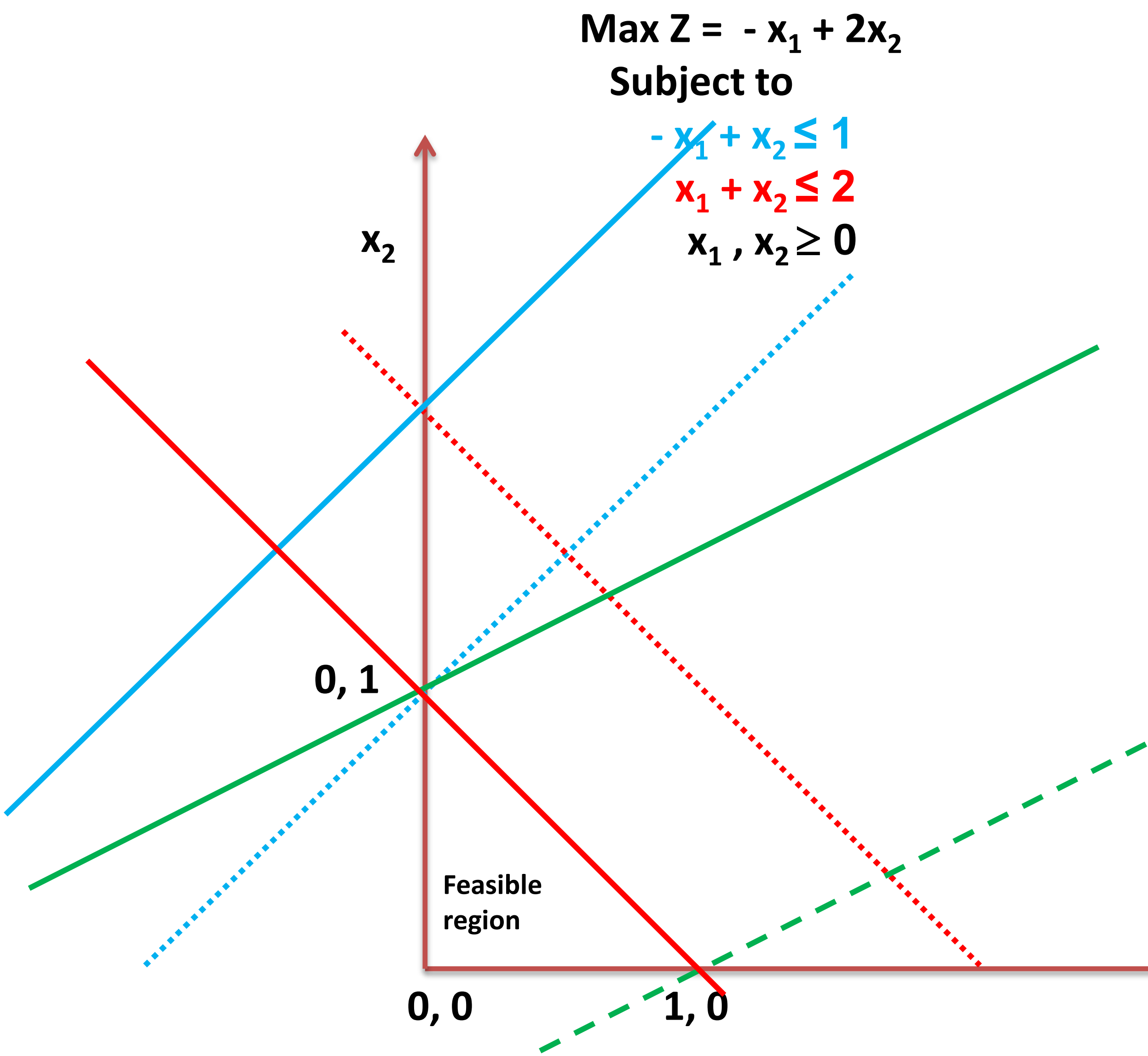
$x_2 = 3/2$

$Z = 5/2$

Find new optimal solution when objective function is $2x_1 + x_2$



Find optimal solution when RHS of the constraints changes from [1, 2] to [2, 1].




**Max** $Z = -x_1 + 2x_2$

**Subject to**

$-x_1 + x_2 \leq 2$

$x_1 + x_2 \leq 1$

$x_1, x_2 \geq 0$

**Optimal solution**

$x_1 = 0$

$x_2 = 1$

$Z = 2$

Solve LPP when variables are integers

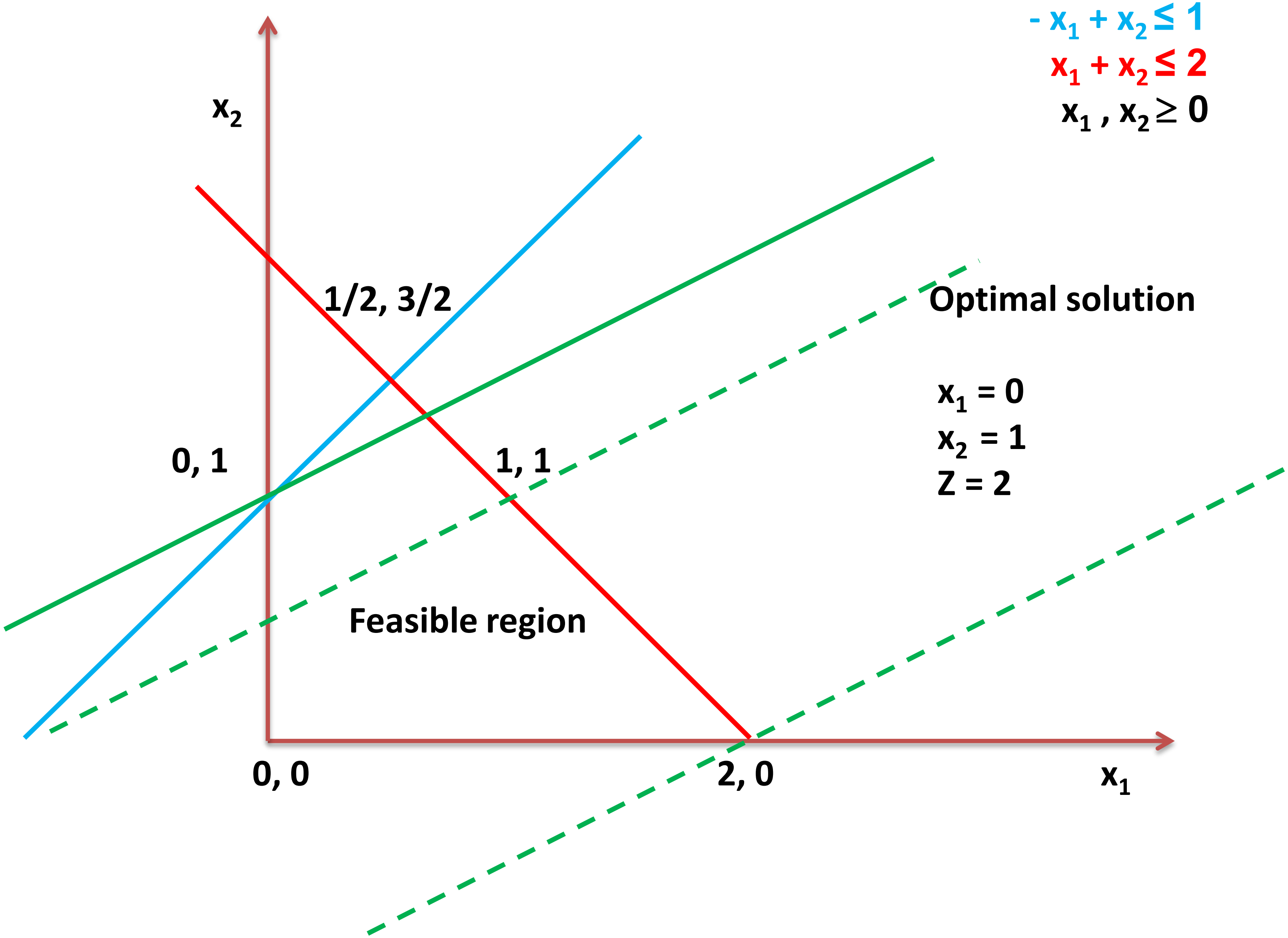
Max $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \le 1$

$x_1 + x_2 \le 2$

$x_1, x_2 \ge 0$




Optimal solution

$x_1 = 0$

$x_2 = 1$

$Z = 2$

Write the dual of the above LPP.

**Max** $Z = -x_1 + 2x_2$

**Subject to**

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

**Min** $W = y_1 + 2y_2$

**Subject to**

$-y_1 + y_2 \geq -1$

$y_1 + y_2 \geq 2$

$y_1, y_2 \geq 0$

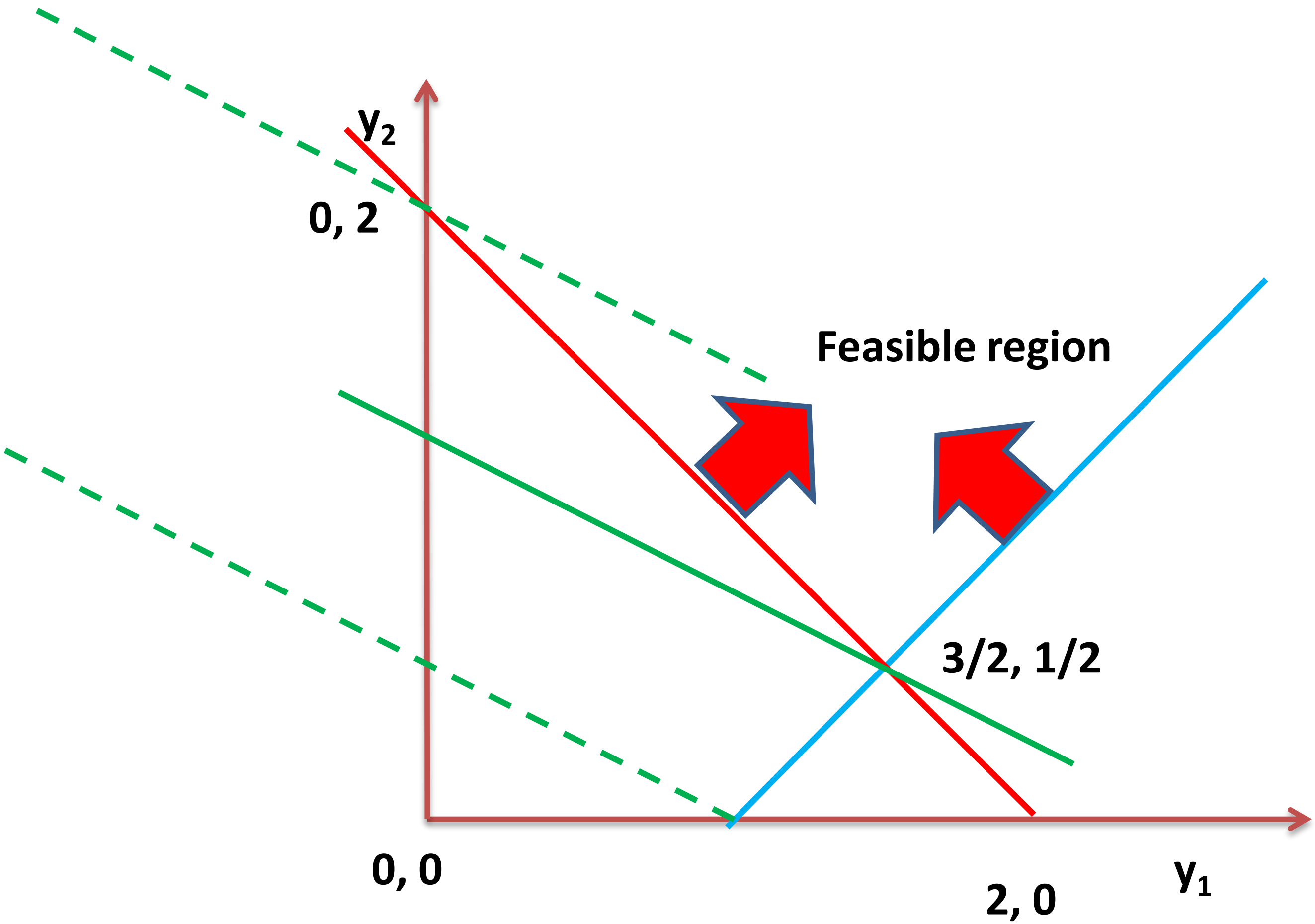
**Min** $W = y_1 + 2y_2$

**Subject to**

$y_1 - y_2 \leq 1$

$y_1 + y_2 \geq 2$

$y_1, y_2 \geq 0$




**Optimal solution**

$y_1 = 1/2$

$y_2 = 3/2$

$W = 5/2$

Max $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

Canonical form

$-x_1 + x_2 + 1.s_1 + 0.s_2 + 0.Z = 1$

$x_1 + x_2 + 0.s_1 + 1.s_2 + 0.Z = 2$

$x_1 - 2x_2 + 0.s_1 + 0.s_2 + 1.Z = 0$

| | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b | Min. ratio |
|---|---|---|---|---|---|---|---|
| R1 | -1 | 1 | 1 | 0 | 0 | 1 | |
| R2 | 1 | 1 | 0 | 1 | 0 | 2 | |
| R3 | 1 | -2 | 0 | 0 | 1 | 0 | |
| R4=R1 | -1 | 1 | 1 | 0 | 0 | 1 | - |
| R5=R2-R4 | 2 | 0 | -1 | 1 | 0 | 1 | 1/2 |
| R6=R3+2R4 | -1 | 0 | 2 | 0 | 1 | 2 | |
| R7=R4+R8 | 0 | 1 | 1/2 | 1/2 | 0 | 3/2 | |
| R8=R5/2 | 1 | 0 | -1/2 | 1/2 | 0 | 1/2 | |
| R9=R8+R6 | 0 | 0 | 3/2 | 1/2 | 1 | 5/2 | |
| | Values of dual variables | | | | | | |

# Find new optimal solution when objective function is $Z = 2x_1 + x_2$

$-x_1 + x_2 + 1.s_1 + 0.s_2 + 0.Z = 1$

$x_1 + x_2 + 0.s_1 + 1.s_2 + 0.Z = 2$

$-2x_1 - x_2 + 0.s_1 + 0.s_2 + 1.Z = 0$

| | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b | Min. ratio |
|---|---|---|---|---|---|---|---|
| R7=R4+R8 | 0 | 1 | 1/2 | 1/2 | 0 | 3/2 | 3 |
| R8=R5/2 | 1 | 0 | -1/2 | 1/2 | 0 | 1/2 | - |
| R9 | 0 | 0 | -1/2 | 3/2 | 1 | 5/2 | |
| R10=2*R7 | 0 | 2 | 1 | 1 | 0 | 3 | |
| R11=R8+R10/2 | 1 | 1 | 0 | 1 | 0 | 2 | |
| R12=R9+R10/2 | 0 | 1 | 0 | 2 | 1 | 4 | |

**Optimal solution**

$x_1 = 2$

$x_2 = 0$

$Z = 4$

Find optimal solution when RHS of the constraints changes from [1, 2] to [2, 1].

**Max** $Z = -x_1 + 2x_2$
**Subject to**
$-x_1 + x_2 \leq 1$
$x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

**Max** $Z = -x_1 + 2x_2$
**Subject to**
$-x_1 + x_2 \leq 2$
$x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0$

| | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b | Min. ratio |
|---|---|---|---|---|---|---|---|
| R7=R4+R8 | 0 | 1 | 1/2 | 1/2 | 0 | 3/2 | |
| R8=R5/2 | 1 | 0 | -1/2 | 1/2 | 0 | 1/2 | |
| R9=R8+R6 | 0 | 0 | 3/2 | 1/2 | 1 | 5/2 | |

Find optimal solution when RHS of the constraints changes from [1, 2] to [2, 1].

Max $Z = -x_1 + 2x_2$
Subject to
$-x_1 + x_2 \leq 1$
$x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

Max $Z = -x_1 + 2x_2$
Subject to
$-x_1 + x_2 \leq 2$
$x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0$

| | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b | Min. ratio |
|---|---|---|---|---|---|---|---|
| R7=R4+R8 | 0 | 1 | 1/2 | 1/2 | 0 | 3/2 | |
| R8=R5/2 | 1 | 0 | -1/2 | 1/2 | 0 | 1/2 | |
| R9=R8+R6 | 0 | 0 | 3/2 | 1/2 | 1 | 5/2 | |

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 3/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 1/2 \\ 5/2 \end{pmatrix}$$

Find optimal solution when RHS of the constraints changes from [1, 2] to [2, 1].

**Max** $Z = -x_1 + 2x_2$
**Subject to**
$-x_1 + x_2 \leq 1$
$x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

**Max** $Z = -x_1 + 2x_2$
**Subject to**
$-x_1 + x_2 \leq 2$
$x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0$

| | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b | Min. ratio |
|---|---|---|---|---|---|---|---|
| R7=R4+R8 | 0 | 1 | 1/2 | 1/2 | 0 | 3/2 | |
| R8=R5/2 | 1 | 0 | -1/2 | 1/2 | 0 | 1/2 | |
| R9=R8+R6 | 0 | 0 | 3/2 | 1/2 | 1 | 5/2 | |

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 3/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 1/2 \\ 5/2 \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 3/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3/2 \\ -1/2 \\ 7/2 \end{pmatrix}$$

Find optimal solution when RHS of the constraints changes from [1, 2] to [2, 1].

| | $X_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b | Min. ratio |
|---|---|---|---|---|---|---|---|
| R7=R4+R8 | 0 | 1 | 1/2 | 1/2 | 0 | 3/2 | |
| R8=R5/2 | 1 | 0 | -1/2 | 1/2 | 0 | -1/2 | |
| R9=R8+R6 | 0 | 0 | 3/2 | 1/2 | 1 | 7/2 | |
| R10=R7+R8 | 1 | 1 | 0 | 1 | 0 | 1 | |
| R11=-2*R8 | -2 | 0 | 1 | -1 | 0 | 1 | |
| R12=3*R8+R9 | 3 | 0 | 0 | 2 | 1 | 2 | |

**Optimal solution**

$x_1 = 0$

$x_2 = 1$

$S_1 = 1$

$Z = 2$

Solve LPP when variables are integers

**Max** $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ and integers

- **Find solution by simplex method.**
- **Apply branch and bound method.**
- **Introduce new constraint in the optimal table.**
- **Convert into canonical form.**
- **Apply dual simplex method to get integer solution.**

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b |
|---|---|---|---|---|---|
| -1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 2 |
| 1 | -2 | 0 | 0 | 1 | 0 |
| -1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | -1 | 1 | 0 | 1 |
| -1 | 0 | 2 | 0 | 1 | 2 |
| 0 | 1 | 1/2 | 1/2 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 1/2 |
| 0 | 0 | 3/2 | 1/2 | 1 | 5/2 |
| | | | | | |

Solve LPP when variables are integers

**Max** $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ **and integers**

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | Z | b |
|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 1/2 |
| 0 | 0 | 3/2 | 1/2 | 1 | 5/2 |

$X_2 \leq 1$

$X_2 + S_3 = 1$

R3 = R3 – R1

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Z | b |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 0 | 1/2 |
| 0 | 0 | -1/2 | -1/2 | 1 | 0 | -1/2 |
| 0 | 0 | 3/2 | 1/2 | 0 | 1 | 5/2 |

$X_2 \geq 2$

$-X_2 \leq -2$

or

$-- X_2 + S_3 = -2$

R3 = R3 + R1

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Z | b |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 0 | 1/2 |
| 0 | 0 | 1/2 | 1/2 | 1 | 0 | -1/2 |
| 0 | 0 | 3/2 | 1/2 | 0 | 1 | 5/2 |

Solve LPP when variables are integers

Max $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ and integers

| $X_1 = 1/2$ | $X_2 = 3/2$ | $Z = 5/2$ |
|---|---|---|

$X_2 \leq 1$

$X_2 \geq 2$

**Infeasible**

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Z | b |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 0 | 1/2 |
| 0 | 0 | -1/2 | -1/2 | 1 | 0 | -1/2 |
| 0 | 0 | 3/2 | 1/2 | 0 | 1 | 5/2 |

**Apply dual simplex**

**Third row is pivot row.**

**Find min[{ (3/2/-(-1/2)}, { (1/2/-(-1/2)}] = [3, 1].**

**So, fourth column is pivot column.**

Solve LPP when variables are integers

**Max** $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ and integers

| $X_1 = 1/2$ | $X_2 = 3/2$ | $Z = 5/2$ |
|---|---|---|

$X_2 \leq 1$

$X_2 \geq 2$

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Z | b |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 0 | 1/2 |
| 0 | 0 | -1/2 | -1/2 | 1 | 0 | -1/2 |
| 0 | 0 | 3/2 | 1/2 | 0 | 1 | 5/2 |

**Infeasible**

**Apply dual simplex**

**Third row is pivot row.**

**Find min[{ (3/2/-(-1/2)}, { (1/2/-(-1/2)}] = [3, 1].**

**So, fourth column is pivot column.**

Solve LPP when variables are integers

**Max** $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ **and integers**

| $X_1 = 1/2$ | $X_2 = 3/2$ | $Z = 5/2$ |
|---|---|---|

$X_2 \leq 1$

$X_2 \geq 2$

**Infeasible**

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Z | b |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 0 | 1/2 |
| 0 | 0 | -1/2 | -1/2 | 1 | 0 | -1/2 |
| 0 | 0 | 3/2 | 1/2 | 0 | 1 | 5/2 |
| | | | | | | |
| | | | | | | |
| 0 | 0 | 1 | 1 | -2 | 0 | 1 |
| | | | | | | |

Solve LPP when variables are integers

**Max** $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ and integers

| $X_1 = 1/2$ | $X_2 = 3/2$ | $Z = 5/2$ |
|---|---|---|

$X_2 \leq 1$

$X_2 \geq 2$

**Infeasible**

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Z | b |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/2 | 0 | 0 | 3/2 |
| 1 | 0 | -1/2 | 1/2 | 0 | 0 | 1/2 |
| 0 | 0 | -1/2 | -1/2 | 1 | 0 | -1/2 |
| 0 | 0 | 3/2 | 1/2 | 0 | 1 | 5/2 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | -1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | -2 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 2 |

Solve LPP when variables are integers

**Max** $Z = -x_1 + 2x_2$

Subject to

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ **and integers**

| $X_1 = 1/2$ | $X_2 = 3/2$ | $Z = 5/2$ |
|---|---|---|

$X_2 \leq 1$ $X_2 \geq 2$

**Infeasible**

| $x_1$ | $x_2$ | $s_1$ | $s_2$ | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 1/2 | 1/ | | | |
| 1 | 0 | -1/2 | 1/ | | | |
| 0 | 0 | -1/2 | -1 | | | |
| 0 | 0 | 3/2 | 1/2 | | | |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | -1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | -2 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 2 |

**Optimal integer solution**

$x_1 = 0$
$x_2 = 1$
$Z = 2$